1

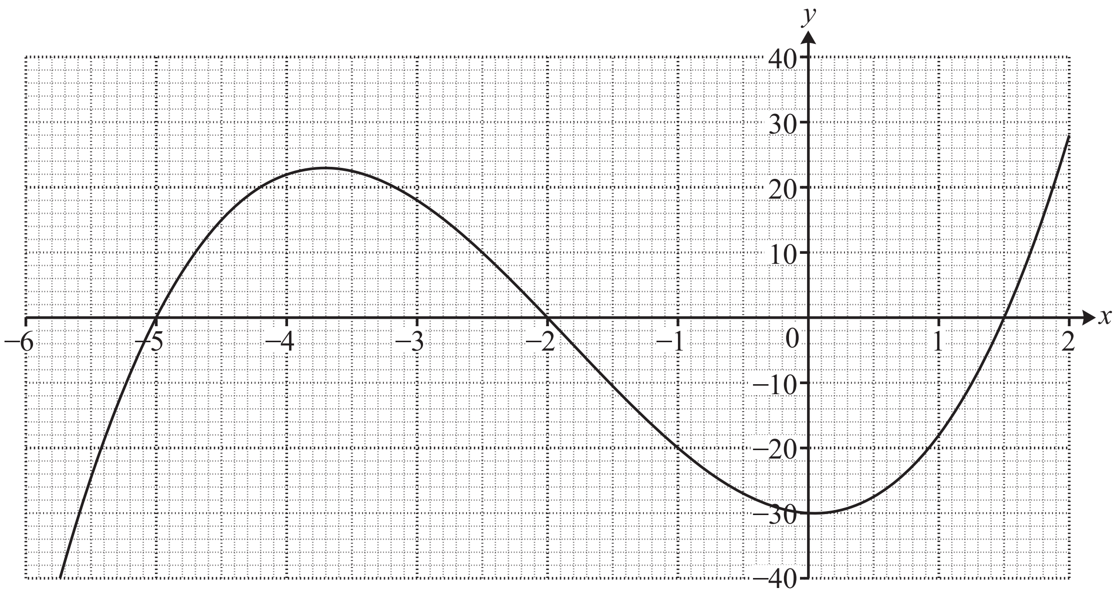

Fig. 12

Fig. 12 shows the graph of a cubic curve. It intersects the axes at (–5, 0), (–2, 0), (1.5, 0) and (0, –30).

(i) Use the intersections with both axes to express the equation of the curve in a factorised form. **[2]**

(ii) Hence show that the equation of the curve may be written as $y = 2x^3 + 11x^2 - x - 30$. **[2]**

(iii) Draw the line $y = 5x + 10$ accurately on the graph. The curve and this line intersect at (–2, 0); find graphically the x-coordinates of the other points of intersection. **[3]**

(iv) Show algebraically that the x-coordinates of the other points of intersection satisfy the equation

$$2x^2 + 7x - 20 = 0.$$

Hence find the exact values of the x-coordinates of the other points of intersection. **[5]**

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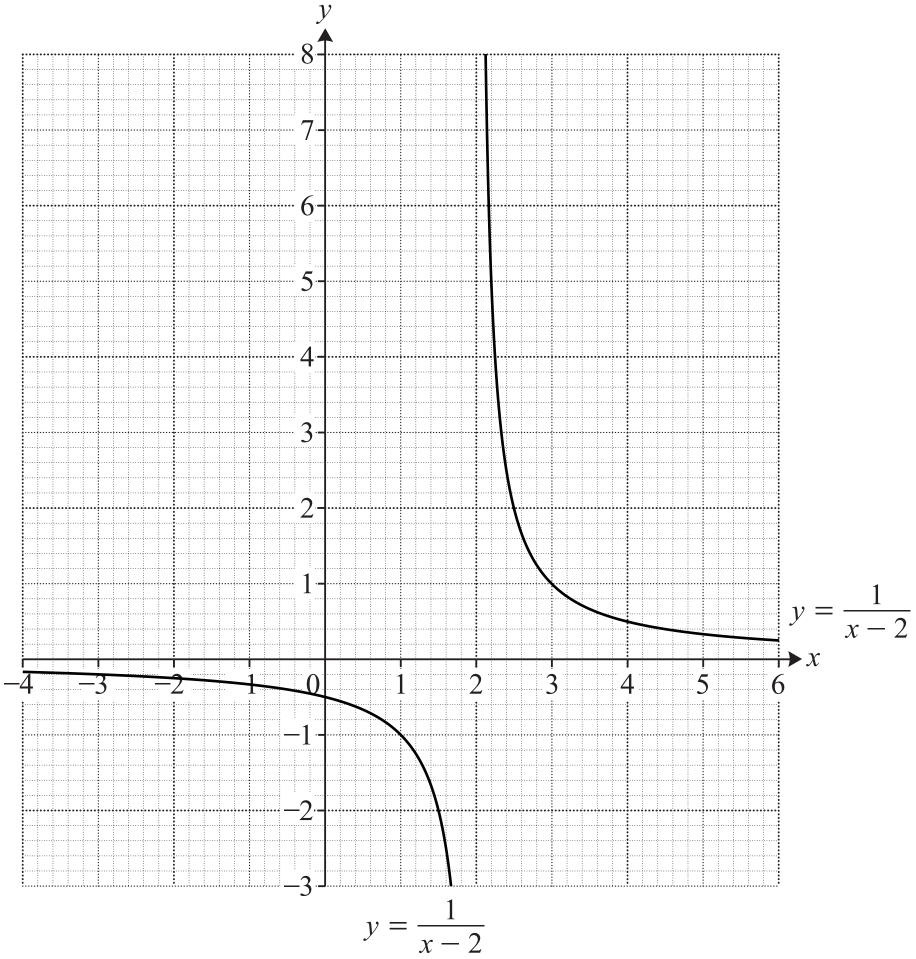

Fig. 12

Fig. 12 shows the graph of $y = \frac{1}{x-2}$.

(i) Draw accurately the graph of $y = 2x + 3$ on the copy of Fig. 12 and use it to estimate the coordinates of the points of intersection of $y = \frac{1}{x-2}$ and $y = 2x + 3$. **[3]**

(ii) Show algebraically that the x-coordinates of the points of intersection of $y = \frac{1}{x-2}$ and $y = 2x + 3$ satisfy the equation $2x^2 - x - 7 = 0$. Hence find the exact values of the x-coordinates of the points of intersection. **[5]**

(iii) Find the quadratic equation satisfied by the x-coordinates of the points of intersection of $y = \frac{1}{x-2}$ and $y = -x + k$. Hence find the exact values of k for which $y = -x + k$ is a tangent to $y = \frac{1}{x-2}$. **[4]**

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3

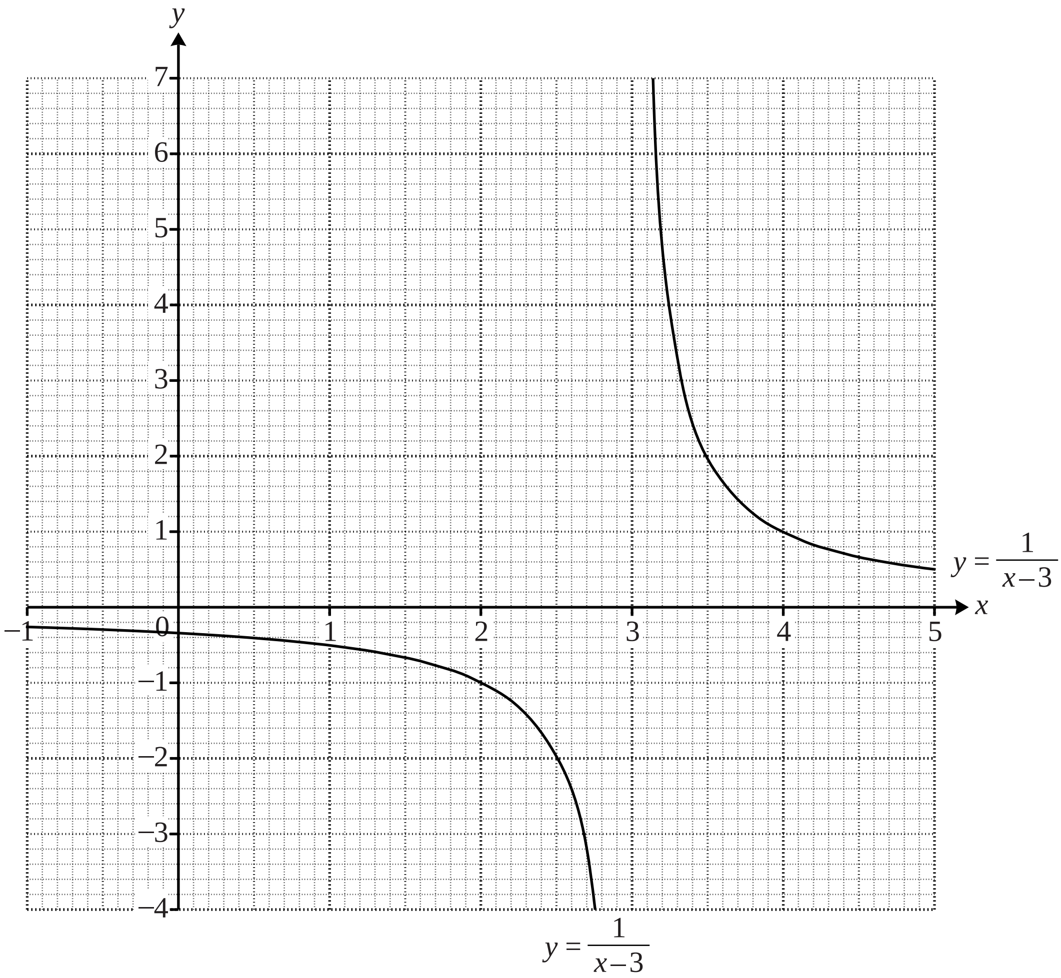

Fig. 12

Fig. 12 shows the graph of $y = \frac{1}{x-3}$.

(i) Draw accurately, on the copy of Fig. 12, the graph of $y = x^2 - 4x + 1$ for $-1 \leqslant x \leqslant 5$. Use your graph to estimate the coordinates of the intersections of $y = \frac{1}{x-3}$ and $y = x^2 - 4x + 1$. **[5]**

(ii) Show algebraically that, where the curves intersect, $x^3 - 7x^2 + 13x - 4 = 0$. **[3]**

(iii) Use the fact that $x = 4$ is a root of $x^3 - 7x^2 + 13x - 4 = 0$ to find a quadratic factor of $x^3 - 7x^2 + 13x - 4$. Hence find the exact values of the other two roots of this equation. **[5]**

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4 **(i)** Find algebraically the coordinates of the points of intersection of the curve $y = 4x^2 + 24x + 31$ and the line $x + y = 10$. **[5]**

(ii) Express $4x^2 + 24x + 31$ in the form $a(x + b)^2 + c$. **[4]**

(iii) For the curve $y = 4x^2 + 24x + 31$,

(A) write down the equation of the line of symmetry, **[1]**

(B) write down the minimum y-value on the curve. **[1]**

5 **(i)** Solve, by factorising, the equation $2x^2 - x - 3 = 0$. **[3]**

(ii) Sketch the graph of $y = 2x^2 - x - 3$. **[3]**

(iii) Show that the equation $x^2 - 5x + 10 = 0$ has no real roots. **[2]**

(iv) Find the x-coordinates of the points of intersection of the graphs of $y = 2x^2 - x - 3$ and $y = x^2 - 5x + 10$. Give your answer in the form $a \pm \sqrt{b}$. **[4]**

6 **Answer the whole of this question on the insert provided.**

The insert shows the graph of $y = \frac{1}{x}$, $x \neq 0$.

(i) Use the graph to find approximate roots of the equation $\frac{1}{x} = 2x + 3$, showing your method clearly. [3]

(ii) Rearrange the equation $\frac{1}{x} = 2x + 3$ to form a quadratic equation. Solve the resulting equation, leaving your answers in the form $\frac{p \pm \sqrt{q}}{r}$. [5]

(iii) Draw the graph of $y = \frac{1}{x} + 2$, $x \neq 0$, on the grid used for part (**i**). [2]

(iv) Write down the values of x which satisfy the equation $\frac{1}{x} + 2 = 2x + 3$. [2]

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